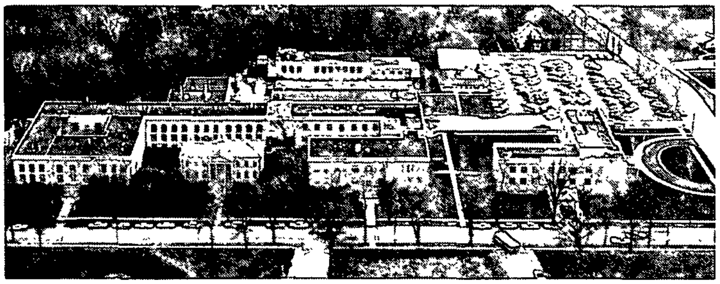

THE INSTITUTE OF PAPER CHEMISTRY, APPLETON, WISCONSIN

IPC TECHNICAL PAPER SERIES
NUMBER 119

PROCEDURES FOR MEASURING THE IN-PLANE ORTHOTROPIC ELASTIC CONSTANTS OF PAPER USING ULTRASONIC TECHNIQUES

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DECEMBER, 1981

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PROCEDURES FOR MEASURING THE IN-PLANE ORTHOTROPIC ELASTIC CONSTANTS OF PAPER USING ULTRASONIC TECHNIQUES

Several recent articles have described the theory and techniques for measuring the nine elastic constants of paper when it is treated as an orthotropic material (1-5). The procedures for measuring the four in-plane elastic constants are given here.

The four in-plane elastic constants, in engineering terms, are the following: Young's moduli in the machine direction (MD) and cross machine direction (CD), E_{MD} and E_{CD}, respectively; the shear modulus G_{MD-CD}; and the Poisson ratio ν_{MD-CD}. These are normally written as E_x, E_y, G_{xy}, and ν_{xy}, respectively. Poisson's ratio, ν_{xy}, is the ratio of the lateral contraction of a specimen in the x-direction to the elongation in the y-direction, when the specimen is stressed uniaxially in the y-direction. Because paper is anisotropic, the Poisson ratio defined when uniaxially stressing the paper in the x-direction, ν_{yx}, is different from ν_{xy}. The two are related by $\nu_{yx} = (E_x/E_y)\,\nu_{xy}$.

The four elastic parameters can be expressed in terms of four measurable velocities. These include the two longitudinal velocities, C_x and C_y in the MD and CD, respectively, a shear velocity in either the MD or CD, C_s, and the shear velocity at 45° to the MD, C_{45}. Longitudinal or shear modes are easily obtained by the orientation of the ultrasonic transducers on the paper, or by use of transducers vibrating in only one mode, as will be described later. The equations relating the elastic constants and velocities are (5)

$$E_x = E_{MD} = \rho C_x^2(1 - \nu_{xy}\nu_{yx})$$

$$E_y = E_{CD} = \rho C_y^2(1 - \nu_{xy}\nu_{yx})$$

$$G_{xy} = G_{MD-CD} = \rho C_s{}^2$$

$$\nu_x = \nu_{xy} = \frac{1}{B}\left(1 + \frac{4B^2}{A^2} - \frac{2B^2}{A} - \frac{2B^2}{AR} - \frac{4B}{A} + \frac{B^2}{R} + \frac{B}{R} + B\right)^{1/2} - \frac{1}{B}$$

where

ρ = apparent density

$B = (C_x/C_s)^2$

$A = (C_x/C_{45})^2$

$R = (C_x/C_y)^2$

$\nu_{yx} = R\nu_{xy}$

The velocities are determined by measuring the transit time of a short burst of sine waves (pulse) through the specimen. The ratio of transducer separation distance to transit time is the velocity. If only a single measurement were made, the transit time would need to be corrected for delays not caused by the paper - an often difficult task. In the procedure described here, this problem is eliminated.

Schematic diagrams for two measuring systems are shown in Fig. 1 and 2. In Fig. 1, the output pulse from the function generator is amplified and fed to the sending transducer. Coincident with the first positive peak in this pulse is the positive slope of the SYNC OUT square pulse. The main time base of the oscilloscope is triggered by this positive slope. At the same time, the scope generates a pulse which starts a time interval counter. The mechanical disturbance transmitted through the particular specimen is converted back to an electrical signal, which is amplified and displayed on the oscilloscope. By adjusting the delay-time multiplier knob on the scope, the instant of triggering of a second (delayed) time base is controlled by the operator. The scope provides visualization of the precise point of triggering. Finally, coincident with the triggering of the delayed time

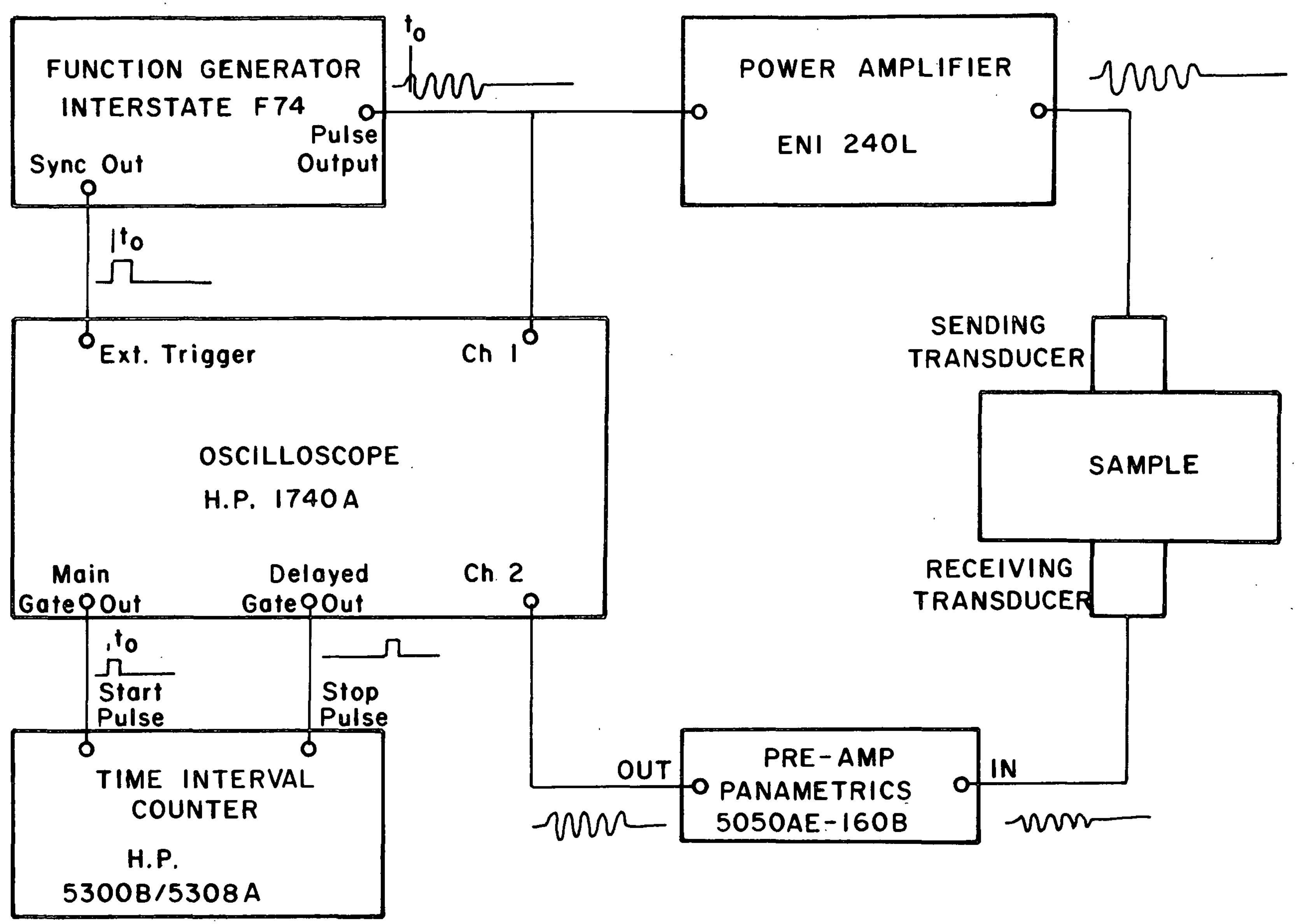

Figure 1. System for making velocity measurements.

base is the delayed GATE OUT which stops the counter. Delay times are measured out to a positive peak in the received signal, typically near the middle of the pulse. Delay time intervals are averaged by the digital display counter.

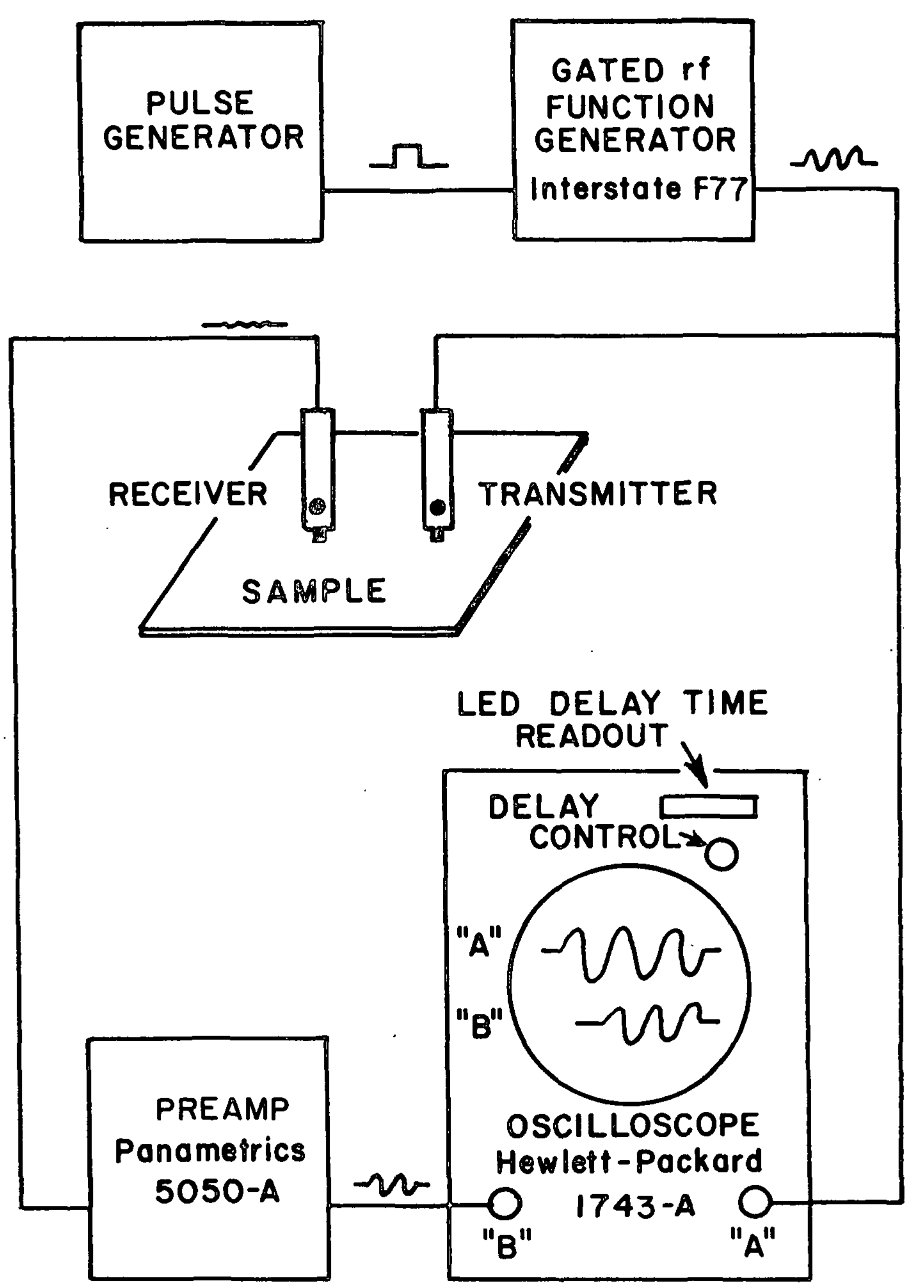

Figure 2. System for making velocity measurements. The dots on the transducers show the direction of polarization. As shown, the transducers would vibrate in a direction perpendicular to a line connecting them, that is, a shear mode vibration.

Figure 2 shows a similar circuit except that the oscilloscope has a delay time feature which eliminates the need for the time interval counter. A pulse generator is shown in conjunction with a function generator to give a burst of sine waves. The time between pulses must be long enough for all mechanical oscillations in the paper to die out. The pulse driving the transmitter is fed into Channel "A" on the oscilloscope. The signal detected by the receiver, after some time delay, is amplified and fed into Channel "B." The delay control on the oscilloscope is then adjusted until a selected part of A and B overlap. The delay time is read directly off the LED display on the oscilloscope.

The procedures used in gathering data are perhaps best understood by referring to Fig. 3, which schematically depicts the paper or board specimen with point contacting transducers (arrows). The transducers are aligned in the machine direction (MD) of the paper but could be oriented in any other in-plane direction. The transducers are polarized, such that either longitudinal or transverse (shear) wave measurements can be made. If the polarization direction of each transducer is also aligned in the MD, then a longitudinal wave is measured in this direction. If the polarization direction of each transducer is oriented 90° to the MD, the situation depicted in Fig. 3 would correspond to a shear wave velocity measurement in the MD.

In all measurements the transducers are positioned some distance (d) apart. The value of d is not critical, except that a distance much less than about one-half wavelength of ultrasound in the paper will result in undesirable interference effects (6). For measurements at, say, 100 kHz for a paper with an MD longitudinal velocity of 3 x 10^3 m/second (typical), the wavelength of the sound is 3 cm. In this case, d should be greater than 3 cm. At lower sound velocities (e.g., shear waves), or as the frequency decreases, the wavelength increases, making the minimum allowable d larger.

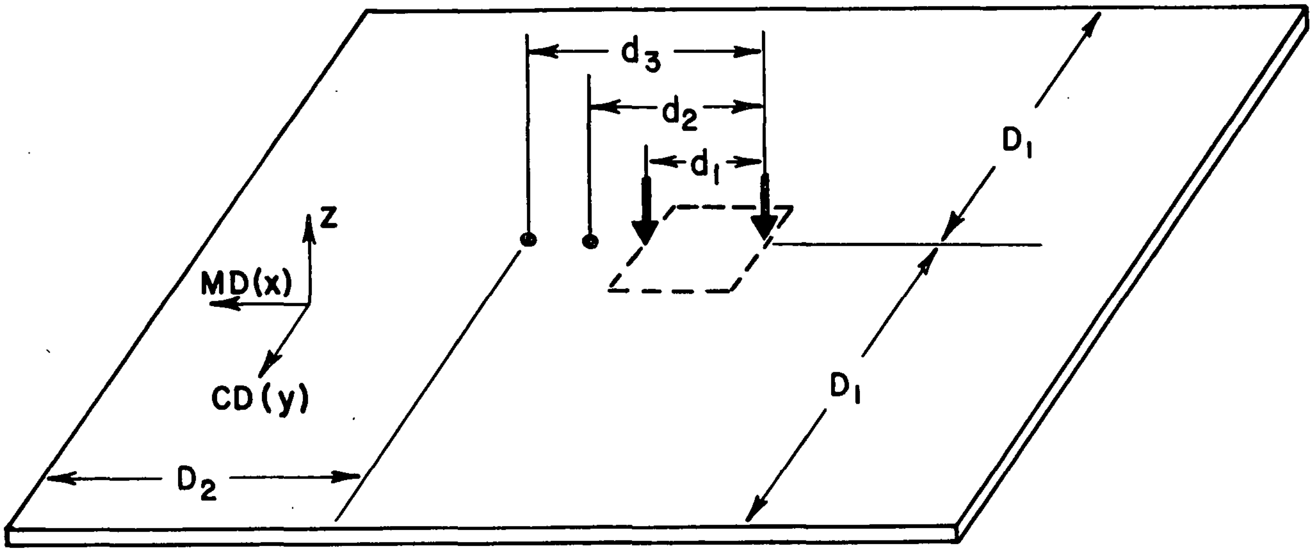

Figure 3. Schematic depicting details of measurements.

The transducers should not be placed too near the edge of the specimen. This will prevent reflections off these boundaries from interfering with the measurements. As a rule of thumb, distances D_1 and D_2 should be greater than the largest separation distance to be used.

During actual measurements, one transducer serves as a transmitter and the other as a receiver. At the separation distance d_1, the transmitter is excited with a burst of sine waves. The time for the mechanical disturbance to reach the receiver is Δt_1. This process can be repeated any number of times with the measured value being displayed on an electronic counter. Our procedure is to average from 100 to 1000 such pulses using the counter. The resultant average Δt is recorded or entered into a hand calculator. Without changing d_1, the transducers are next repositioned parallel to the previous measurement path, but from 1 to 3 mm away from it. An average Δt is again obtained. This procedure is repeated at least 10 times (usually by pulling the paper slightly beneath the transducers), and the average values of Δt at each setting are themselves averaged. This single value of Δt is taken as the transit time associated with the separation distance d_1. The dotted lines on Fig. 3 depict the area of sample measured by this procedure.

The transducers are next separated to distance d_2 and the entire procedure repeated. At least three data pairs (d, Δt) are obtained in this way, keeping all of the measurements in approximately the same region of the specimen. A linear regression is then performed on the data pairs, treating Δt as the independent variable. The slope of the resulting line is the velocity of sound propagation in the paper. The data should fall on a straight line with a correlation coefficient of at least 0.999. Values less than this are unusual (say 0.997) and suggest experimental or operator errors. If a calculator is not available to perform the regression, the transit times may be plotted as functions of d. The reciprocal slope of this line is the velocity ($\Delta d/\Delta t$). By obtaining transit times at several separation distances and finding the slope of distance vs. time, the need to correct Δt for nonpaper delays is avoided. (The intercept time at an extrapolated zero separation distance is a measure of the nonpaper delays caused by delays in the electronics, cables, transducers, etc.)

To obtain all four independent elastic constants in the plane of the sheet, it is necessary to measure four different velocities. These include the two longitudinal velocities, C_x and C_y in the MD and CD, respectively, a shear velocity in either the MD or CD (same velocity) (C_s), and the shear velocity at 45° to the MD (C_{45}). The portion of the specimen viewed in each of these measurements is shown in Fig. 4. The four measurements are made such that the same general part of the specimen is viewed in each.*

*In the method described, a new portion of the specimen is included in the measurement each time d is increased (Fig. 3). An alternate method which avoids this is to make the measurements at the longest separation distance first, and then to make the measurements at shorter separation distances at random locations throughout the area defined by the first measurements.

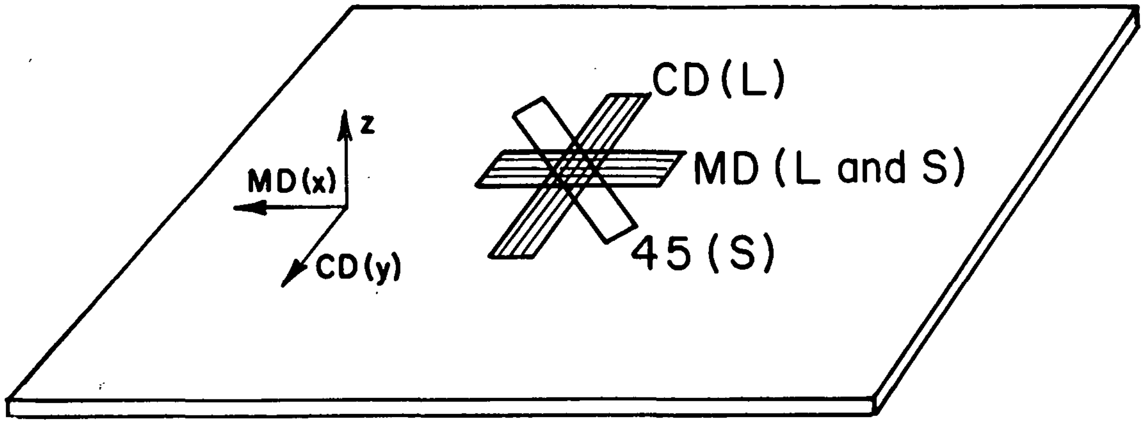

Figure 4. All four measurements should be made in the same region of the specimen. L = longitudinal mode, S = shear mode. Forty-five represents 45° to the MD.

After calculating the four velocities (as defined earlier), the four engineering elastic constants in the plane of the paper are calculated according to the equations given previously.

It is necessary to measure the apparent density to calculate the engineering elastic constants. This will introduce an error due to errors in the measurement of caliper. This can be avoided if one can use "specific stiffnesses," rigorously defined in terms of the linear theory of elasticity. See, for example, references 1-4.

Transducers of the type shown in Fig. 2 or 3, together with a platform which allows measurement of the transducer separation distance, are commercially available (7).

As in any measurement of paper properties, testing should be carried out in standard conditions, with appropriate preconditioning. Our procedure is to test at

50% RH, 73°F (23°C), after preconditioning at 20% RH, 73°F (23°C), unless different moisture levels are sought.

Table I gives some typical results which may be useful for comparison. A relationship between the in-plane constants has been discovered which may also provide a check of the data. The quantity $(\upsilon_x\upsilon_y)^{1/2}$ appears to be relatively insensitive to apparent density, to furnish, to the method used to form the sheet, and to moisture content. The mean value of $(\upsilon_x\upsilon_y)^{1/2}$ for many samples is about 0.293, with a standard deviation of 0.023. This surprising discovery has led to a relationship

$$G_{xy} = 0.387\ (E_x\ E_y)^{1/2},$$

where the constant is calculated from the measured Poisson ratios. The relationship is believed to apply to anisotropy ratios, R, less than about 3.

Work is currently under way in our laboratories to measure the elastic properties in the z-direction or thickness direction. Such measurements are more difficult than the in-plane measurements because paper is so thin. We hope to update this list of procedures to include z-direction measurements. The equipment shown in Fig. 1 and 2 is suitable for such measurements, but special transducers and sample holders are required.

TABLE I

TYPICAL DATA[a]

	Basis Weight, g/m^2	Density, kg/m^3	E_x, GPa	E_y, GPa	G_{xy}, GPa	υ_{xy}	υ_{yx}	R, (E_x/E_y)
Handsheet, unbleached kraft SW	143	439	2.50	2.65	0.99	0.310	0.293	0.99
Handsheet, unbleached kraft HW	214	658	5.20	5.20	1.99	0.306	0.306	1.00
Linerboard, 20% RH	330	644	7.36	2.88	1.74	0.177	0.453	2.55
Linerboard, 35% RH	333	638	6.82	2.70	1.65	0.192	0.485	2.52
Linerboard, 50% RH	337	639	6.36	2.48	1.57	0.176	0.452	2.57
Linerboard, 65% RH	350	632	5.55	2.09	1.32	0.184	0.487	2.65
Linerboard, 80% RH	357	619	4.99	1.78	1.15	0.186	0.523	2.81
Linerboard	50	762	10.77	5.22	2.94	0.206	0.425	2.07
Linerboard	124	512	5.02	1.93	1.21	0.171	0.443	2.59
Linerboard	219	702	6.89	2.15	1.52	0.170	0.544	3.21
Linerboard	338	634	6.80	2.93	1.77	0.194	0.452	2.32
Linerboard	406	633	6.75	3.09	1.83	0.221	0.483	2.19
Medium	168	550	4.56	1.98	1.19	0.172	0.403	2.34
Boxboard	494	750	7.24	2.19	1.54	0.164	0.541	3.32
Carton stock	320	736	5.85	3.08	1.69	0.199	0.378	1.90

[a]All measurements at 50% RH, 73°F (23°C), unless otherwise noted.

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